

ON PURPOSE AND FORMULATION OF NORMATIVE MODELS: A FREE ENERGY PERSPECTIVE

James Hubert Merrick¹

March 9th, 2021

Abstract

This paper explores whether the Free Energy Principle has implications for the formulation and use of normative models. These models are used for policy analysis, strategic planning, system analysis in areas as diverse from pandemic response and avoidance to climate change.

GEAL RESEARCH WORKING PAPER SERIES

GR-01-v1. The Geal Research Working Paper Series enables discussion and dissemination of ideas while work is still ongoing. Comments very welcome.

©2021. This manuscript version is made available under the CC-BY-NC-ND 4.0 license
<http://creativecommons.org/licenses/by-nc-nd/4.0/>

¹james@geal.ie, jmerrick@alumni.stanford.edu

1 Introduction

This paper investigate the purpose and formulation of normative models, and whether a principle entitled the Free Energy Principle may be used to improve the state of the art. Normative models, as the term is used here, refers to models that support decisions for individuals or organisations. Models based on mathematical optimization we will especially consider. Choosing the right model formulation for a given question is traditionally in the realm of colloquial good practice, with a lack of underlying theory. This approach leads to guidelines, often very sensible, for appropriate model formulation and use, such as that provided by [Saltelli et al. \(2020\)](#).

The Free Energy Principle states “that any self-organizing system that is at equilibrium with its environment must minimize its free energy” ([Friston, 2010](#)). And what is free energy? It was originally developed as a thermodynamic concept in the 19th century, that was since generalized to a broader statistical concept, which in turn opens it up for multiple applications and interpretations. In addition to offering a useful background to the concept, [Gottwald and Braun \(2020\)](#) offer two helpful interpretations in the context for decision making: a) normative rule for action selection b) active inference. We will explore what both interpretations imply for the formulation of our models.

2 Background

[Merrick and Weyant \(2019\)](#) discuss the challenges with validation of normative models. and mapped a subset of the model formulation problem, that of choosing model resolution, to the information bottleneck idea of [Tishby et al. \(1999\)](#), trading off complexity and accuracy. Noteworthy is that this Information Bottleneck idea has been shown to be mathematically equivalent to the Free Energy principle ([Friston, 2012](#)).

Free Energy is treated mathematically in a comprehensive fashion by earlier references. For the purposes of our discussion we can represent its essence informally as:

$$\text{energy} - \beta(\text{entropy}) \tag{1}$$

That is energy tempered by entropy, with the relative importance set by some constant β .

3 Two free energy interpretations relevant for modeling

The purpose of a normative model may be summarized by the statement of [Weyant \(2020\)](#): “*Models identify challenges and opportunities in a useful way that is actionable for stakeholders*”. The following two interpretations of free energy principle enable this / support this in different ways.

3.1 Normative rule for action selection

Under this interpretation, the energy term in free energy can be interpreted as cost or utility, while entropy can be considered a representation of uncertainty. A normative rule for action selection is

to choose actions that tradeoff utility / cost of an action against uncertainties.

$$\text{Cost} - \beta(\text{uncertainty}) \tag{2}$$

There is an interesting connection to the work of Ghadimi et al. (2020) and related literature - where it is shown that a characterization of uncertainty can be represented in a deterministic model under certain conditions, and that numerous intuitive ‘robustness’ constraints typically deployed in models used in practice can be interpreted as such a representation of uncertainty.

A further interpretation is maximizing accuracy subject to a tradeoff with complexity, leading to the Information Bottleneck of Tishby et al. (1999), which in turn was applied by Merrick and Weyant (2019) to consider normative model resolution.

This interpretation of the free energy principle provides a guide for model formulation, by highlighting the need to avoid brittle solutions, or the recommendation of actions that have narrow optima, that may turn out to be very bad outcomes under some fluctuation between reality and assumptions.

3.2 Active inference

Under this interpretation, Free Energy to be minimized is:

$$d(\text{reality, internal model}) - \beta(\text{change reality through action}) \tag{3}$$

with $d()$ a distance function, while β again represents the relative cost of the two terms. We can consider actions to be in the loop of the decisionmaking, and free energy minimization to be a minimization of surprise between reality and an agent or entities internal model. To minimize this quantity, there are two choices, change one’s internal model or change reality through action.²

A normative model can help with first term above, helping to reduce gap between internal model and reality, by aiding understanding.³ And a normative model may also help with assessment of the second term, illustrating how reality may change action. This fits within the goal of identifying challenges and opportunities that are actionable for stakeholders.

4 Why Free Energy?

Importantly, if the minimization of free energy is a useful concept for modeling an oil drop in a bucket to the organisation of the brain, is it also useful for moving further up the scales to organisational and societal scales? This latter view is deemed appropriate by Özkural (2019).

²There is a connection to reinforcement learning, with action in the loop. For example, the DeepMind $\mu 0$ model (Schrittwieser et al., 2019) has the property of performing expertly well on a range of games without knowing the rules, but by learning them through ‘learning by doing’.

³Bertsimas and Stellato (2020) introduce a new idea to interpret optimization models, which ties in to the overall goal of minimizing surprises by enabling understanding of the model space. The idea is that you train a machine learning model to predict optimization model outputs given optimization model inputs, trained on a dataset of optimization model inputs and outputs. If the machine learning model is a decision tree, you then obtain an interpretable structure that is a useful complement to traditional sensitivity analysis.

5 To conclude

An exploration, with some intriguing connections. Minimizing free energy may be (a) a reasonable way to codify the goals of organisations or societies that normative models model and thus (b) a useful benchmark to guide model formulation.

References

- Bertsimas, D. and Stellato, B. (2020). The Voice of Optimization. *Machine Learning*.
- Friston, K. (2010). The free-energy principle: a unified brain theory? *Nature Reviews Neuroscience*, 11:127–138.
- Friston, K. (2012). A Free Energy Principle for Biological Systems. *Entropy*, 14:2100–2121.
- Ghadimi, S., Perkins, R. T., and Powell, W. B. (2020). Reinforcement learning via parametric cost function approximation for multistage stochastic programming. *arXiv math.OA*, 2001.00831.
- Gottwald, S. and Braun, D. (2020). The two kinds of free energy and the Bayesian revolution. *PLoS Computational Biology*, 16(12):e1008420.
- Merrick, J. H. and Weyant, J. P. (2019). On choosing the resolution of normative models. *European Journal of Operational Research*, 279(2):511–523.
- Özkural, E. (2019). Epistemological and Ethical Implications of the Free Energy Principle. *Preprints*, 2019080318.
- Saltelli, A., Bammer, G., Bruno, I., and et al. (2020). Five ways to ensure that models serve society: A manifesto. *Nature*, 582:482–484.
- Schrittwieser, J., Antonoglou, I., Hubert, T., Simonyan, K., Sifre, L., Schmitt, S., Guez, A., Lockhart, E., Hassabis, D., Graepel, T., Lillicrap, T. P., and Silver, D. (2019). Mastering Atari, Go, Chess and Shogi by Planning with a Learned Model. *Computing Research Repository*, abs/1911.08265.
- Tishby, N., Pereira, F. C., and Bialek, W. (1999). The information bottleneck method. In *The 37th Allerton Conference on Communication, Control, and Computing*.